

Autumn Semester Examination 2018
Paro College of Education
Royal University of Bhutan
Paro

Module : MAT 306 (Analytic Geometry)

Programme: B.Ed(S)

Level : III

Writing Time: Three Hours

Full Marks: 100

Instructions : Do not write during the first 15 minutes. Use this time for reading the questions. You will get three hours for answering the questions. Write the answers to all the questions in the answer sheets provided by the college. Read the directions to each section and to each question carefully before answering the questions. You are allowed to carry a scientific calculator *fx-82 or fx-100* beside other writing materials. You will be provided with graph sheets.

SECTION A
One Question - 40 Marks

Question 1

Instruction: Attempt all sub-questions. Each sub-question carry 4 marks each.

- a. How many planes of symmetry and order of turn symmetry does each shape have?
 - i. Regular Pentagon-based Pyramid
 - ii. Cylinder
 - iii. Square-based pyramid
 - iv. Regular Hexagon-based prism
- b. Using matrix transformation, show that a reflection in the origin give the same result as a reflection in the x-axis followed by a reflection in the y-axis.
- c. Find the image of a triangle ABC which undergoes a translation through a distance $3\sqrt{2}$ units inclined at an angle of $\frac{\pi}{4}$ radians to the x-axis when the co-ordinates of A, B, C are $(0,0), (4,0), (0,4)$ respectively.
- d. Find the equation of the circle which passes through the origin and touches x-axis and y-axis at the point $(3,0)$ and $(4,0)$.
- e. Show that the points $(-2,6), (5,3), (-1,-11)$ and $(-8,-8)$ are the vertices of a rectangle.
- f. Find the locus of a point in which the sum of the squares of its distances from the points $(2,4)$ and $(-3,-1)$ is 30.
- g. What is degenerated conic? Explain different degenerated conics with respect to the angle of plane and generators with axis.
- h. Which conics are represented by the curves given below? Justify your answer.
 - i. $x^2 - 16xy - 11y^2 - 12x + 6y + 21 = 0$;
 - ii. $16x^2 + 8xy + y^2 - 74x - 78y + 212 = 0$
- i. If (l_1, m_1, n_1) and (l_2, m_2, n_2) are the direction cosines of two given lines, prove that $\cos \theta = l_1l_2 + m_1m_2 + n_1n_2$, where θ is the angle between the two lines.
- j. Find the ratio in which the xy-plane divides the join of $(-3,4,-8)$ and $(5,-6,4)$. Also obtain the point of intersection of this line with the plane.

SECTION B
Four Questions - 60 Marks

Instructions: There are SIX questions in this section. Attempt only FOUR questions. Each question carries 15 marks. You must show all working steps for each question.

Question 2

- a. Let $A(6, 3)$, $B(5, 5)$, $C(8, 3)$, $D(5, 1)$ be the vertices of a quadrilateral $ABCD$ and $A'(-2, 3)$, $B'(-4, 2)$, $C'(-2, 5)$, $D'(0, 2)$ be the vertices of a quadrilateral $A'B'C'D'$. [8]
- i. If $A'B'C'D'$ is an image of $ABCD$, graphically find the center and angle of rotation.
- ii. Using the center of rotation as the center of dilatation, find the image graphically for $ABCD$ when scale factor is 1.5.
- iii. Using mapping notation, verify answer (b) using anyone of the vertex.
- b. State and prove the theorem of reflection in the line $y = x \tan \theta$. Using the proved theorem, find the position of point (x, y) when it is reflected in the straight line $y = x$, whose $\theta = \frac{\pi}{4}$ [7]

Question 3

- a. Use mapping notation to answer the following sub-questions: [8]
- i. If $A'(1, 4)$ is an image of the point $A(-1, 2)$ while rotating 90° , find its center of rotation.
- ii. If $B'(4, 4)$ is an image of the point $B(0, 7)$ while dilated with center $(8, 1)$, find dilatation factor.
- iii. If $C'(-3 + 2\sqrt{2}, 3)$ is an image of the point $C(-1, 1)$ while rotated around the point $(-3, 3)$, find its angle of rotation.
- iv. Write a composite notation if the point (x, y) is stretched by factor 2 with y-axis as an invariant line followed by reflection in the line $y = x$.
- b. Find the equation of the circle which passes through the points $(1, -2)$, $(4, -3)$ and has its centre on the line $3x + 4y + 10 = 0$. Find the center and radius of the circle. [7]

Question 4

- a. Find the equation of the line through the intersection of $x + y - 9 = 0$ and $2x - 3y + 7 = 0$, and perpendicular to the line $2y - 3x - 5 = 0$. [7]
- b. State and prove the condition of the angles between two lines when their slopes are given. Using the proved conditions, find the parallel and perpendicular conditions of two lines. [8]

Question 5

- a. Find the equation of a parabola with vertex at $(-2, 3)$ and focus at $(1, 3)$. [7]
- b. Find the centre, vertices, foci, eccentricity, latus rectum, equation of latus rectum, equation of directrix and axes of the ellipse $16x^2 + 25y^2 - 32x - 200y + 16 = 0$. [8]

Question 6

- a. Find the general equation of a conic when focus, directrix and eccentricity are given. Based on the equation, state the condition of each conics. [8]
- b. Find the equation of the hyperbola whose foci are $(0, \pm \sqrt{10})$ and which passes through $(2, 3)$. [7]

Question 7

- a. A, B, C are the points $(1, 4, 2), (-2, 1, 2), (2, -3, 4)$. Find the angles of the triangle ABC and show that it is right angled triangle. [7]
- b. The mid-points of the sides of a triangle are $(1, 5, -1), (0, 4, -2)$ and $(2, 3, 4)$. Find its vertices. [8]